Efficient Quadratic Programming for Peak-to-Average Power Ratio Reduction in Communication Systems

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***Abstract*—In this paper, we propose an enhanced peak-to-average power ratio (PAPR) reduction framework for MIMO-OFDM system based on unimodular quadratic programming (UQP). In addition, we consider a more general setting for PAPR reduction problem in MIMO-OFDM systems and propose a novel power method-like algorithm to effectively tackle the associated UQP. The proposed method can handle arbitrary peak-to-average-power ratio (PAPR) constraints on the transmit sequence, and more importantly, can be used to generate constant modulus signals for such systems. The proposed algorithm demonstrates an improvement in terms of convergence rate compared with the state-of-the-art PAPR reduction method.**

***Index Terms*—Multiple Input Multiple Output (MIMO), beamforming, peak-to-average power ratio (PAPR), unimodular quadratic programming (UQP), convex and non-convex optimization, constant modulus signal design, orthogonal frequency division multiplexing (OFDM)**

I. INTRODUCTION

MIMO-OFDM has attracted a lot of attention from wireless research community and has become one of the most promising techniques for the next generation wireless systems requiring high-speed data rates. Such systems, on the other hand, suffer from disadvantages of the OFDM technique, e.g., sensitivity to time and frequency synchronization error, and the large peak-to-average power ratio (PAPR) of transmitted OFDM signals. In particular, the high PAPR in MIMO-OFDM systems is exacerbated as the number of antennas increases [1]. On the receiver side, the performance of non-linear equipment such as high power amplifier (HPA) and digital-to-analog (DAC) converters can be severely degraded due to high PAPR. For instance, it is desirable to keep the HPA's operation region near saturation to achieve maximum efficiency, while a high PAPR would introduce non-linear distortion in the communication channel and devices which further results in a drastic increase of the error rate at the receiver. Hence, it is crucial to develop PAPR reduction techniques for MIMO-OFDM systems to increase their efficiency in handling large data-streams and further reduce their error rates.

There has been extensive research on PAPR reduction techniques in OFDM systems, e.g., Partial Transmit Sequence (PTS) [2], Active Constellation Extension (ACE) [3], Selected Mapping (SLM) [4] are among the most notable works. In addition, the authors in [5] have proposed a joint PAPR reduction framework for MIMO-OFDM systems. However, most of these method are studied in the context of single-input single-output (SISO) systems and their extension to MIMO systems are not easily applicable, and moreover, their computational complexity are extremely high. More recently, a new promising precoding PAPR reduction method called CP-PTS has been proposed in [6] in MIMO-OFDM systems, where each OFDM block with large number of subcarriers is grouped into Resource Blocks (RB). Then, a complex weighting matrix is assigned to the phase of each RB, and these precoding weighting matrices are then optimized to reduce the PAPR of the transmitted OFDM symbol using either a Steepest Descent Constant Modulus Algorithm (SDCMA) or an alternate Unit-Circle CMA (UC-CMA) [7], [8]. In general, precoding is an effective way to reduce PAPR in OFDM systems. Furthermore, to ensure that the BER performance of the system is not affected by the precoding matrix, it is of importance to design the precoding matrix in a way that the weights lies on the unit circle. An interested reader may consult with [9] and the references therein for further information on PAPR techniques in OFDM signals.

In this paper, we propose an efficient alternative approach to the constant modulus MIMO-OFDM PAPR reduction algorithm in [8]. We first formulate the PAPR reduction in OFDM systems as a Unimodular Quadratic Program (UQP), and then, we employ power method-like iterations for local optimization of the proposed UQP. Our proposed algorithm consider a doubly PAPR constrained scenario in OFDM schemes and can be compared to the state-of-the-art SDCMA and UC-CMA algorithms proposed in [8] in terms of *convergence rate* (i.e., converging faster) with an emphasis on the first.

II. SYSTEM MODELS

In this paper, we undertake a similar setting to [8], and consider an ordinary MIMO-OFDM downlink communication system with one base station (BS) employing N_t transmission antennas. In this setting, each OFDM block consists of N subcarriers to be transmitted from antennas. These N subcarriers combine N_u useful subcarriers each of which surrounded by two zero-energy guard bands (GBs). Moreover, these N_u subcarriers are divided into a set of M resource blocks (RBs) each consisting of $N_b = N_u/M$ subcarriers. Then, each RB is allocated with users data using space-time block coding (STBC) and inverse discrete Fourier transform (IDFT). In addition, several training pilot subcarriers are placed in RBs

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for the purpose of channel estimation at the User Equipment (UE).

We first consider the frequency domain representation of the MIMO transmit signal model. Let $\boldsymbol{D}_r \in \mathbb{C}^{N_t \times N_b}$ denote the r-th RB transmit data model. Then, a corresponding beam-forming matrix $\boldsymbol{W}_r \in \mathbb{C}^{N_t \times N_t}$ (for $r = 1, \ldots, M$) is formed to linearly precode $\boldsymbol{D}_r$ to construct a transmit sequence as

$$\boldsymbol{X}_r = \boldsymbol{W}_r \boldsymbol{D}_r, \tag{1}$$

where $\boldsymbol{X}_r \in \mathbb{C}^{N_t \times N_b}$ is the precoded vector associated with the r-th resource block. Furthermore, let $\boldsymbol{W} = [\boldsymbol{W}_1^H, \ldots, \boldsymbol{W}_M^H]^H$, and $\boldsymbol{D} \in \mathbb{C}^{MN_t \times N}$ be a block-diagonal matrix of the form

$$\boldsymbol{D} = \begin{bmatrix} \boxed{\mathbf{GB}}\ \boxed{\boldsymbol{D}_1} & & & \\ & \boxed{\boldsymbol{D}_2} & & \\ & & \ddots & \\ & & & \boxed{\boldsymbol{D}_M}\ \boxed{\mathbf{GB}} \end{bmatrix}, \tag{2}$$

where $\mathbf{GB}$ denotes the zero-energy subcarriers guard bands. The aggregated data model for a single time block can be then simply modeled as

$$\boldsymbol{X} = \boldsymbol{W}^H \boldsymbol{D}, \tag{3}$$

where N_t rows of the data matrix $\boldsymbol{X} \in \mathbb{C}^{N_t \times N}$ define the N symbols to be transmitted from N_t antennas at the BS. Note that the spatial data in the frequency domain is represented in the data matrix $\boldsymbol{X}$ [8]. After employing the beam-forming via the complex weight matrix $\boldsymbol{W}$, the time-domain representation of the spatial data in the matrix $\boldsymbol{X}$ can be obtained via performing the IDFT operation, viz.

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{F}^H = \boldsymbol{W}^H \underbrace{\boldsymbol{D}\boldsymbol{F}^H}_{\triangleq \boldsymbol{B}}, \tag{4}$$

where $\boldsymbol{F}^H \in \mathbb{C}^{N \times N}$ denotes the IDFT matrix, and the full matrix $\boldsymbol{B}$ is the time-domain representation of the data matrix. More compactly, the beam-formed MIMO-OFDM transmit data can be modeled as $\boldsymbol{Y} = \boldsymbol{W}^H \boldsymbol{B}$.

Let $\boldsymbol{p} = \mathrm{vec}(\boldsymbol{D})$, where $\mathrm{vec}(\cdot)$ denotes the column-wise vectorization operator. Then, the total power of the data matrix $\boldsymbol{D}$ is given by

$$P = \|\boldsymbol{D}\|_F^2 = \|\boldsymbol{p}\|_2^2 = \alpha N_T, \tag{5}$$

where α denotes the average transmit power per sample, and $N_T = N_t N$. Note that if the beam-forming matrices $\{\boldsymbol{W}_{\boldsymbol{r}}\}_{r=1}^{M}$ lies on the Stiefel manifold $\mathrm{St}(N_t, N_t) := \{\boldsymbol{U} \in \mathbb{C}^{N_t \times N_t} | \boldsymbol{U}^H \boldsymbol{U} = \boldsymbol{I}\}$, i.e., $\boldsymbol{W}$ consists of orthonormal matrices; then, $P_Y := \|\boldsymbol{Y}\|_F^2 = \mathrm{Tr}(\boldsymbol{W}^H \boldsymbol{W}) \|\boldsymbol{F}^H\|_F^2 \|\boldsymbol{D}\|_F^2 = P$, where $\mathrm{Tr}(\cdot)$ denotes the trace operator, and thus, the beam-forming and IDFT operation does not affect the total transmit power P. However, as a large number of subcarriers are added with the same phase, the OFDM symbols generates a high peak power in time-domain, which drastically degrades the performance of the communication system. In general, the peak-to-average power ratio of a discrete-time OFDM signal for a MIMO-OFDM block is defined as

$$\mathrm{PAPR}(\boldsymbol{Y}) = \frac{N_T \|\mathrm{vec}(\boldsymbol{Y})\|_\infty^2}{\|\mathrm{vec}(\boldsymbol{Y})\|_2^2} \tag{6}$$

Interestingly, note that the PAPR in (6) assumes it lowest value only when the signal is constant modulus.

In [8], the authors formulate the problem of PAPR reduction as designing a precoding matrix to convert the OFDM symbols in $\boldsymbol{Y}$ to achieve a desirable and ideally constant modulus signal $\boldsymbol{S}$ with lower PAPR than that of the original symbols. In order to do so, they premultiply each $\boldsymbol{D}_r$ with a diagonal complex weighting matrix $\boldsymbol{A}_r$ (this scaling manifests itself as a fading channel to the receiver). In addition, if we further assume that this scaling matrix is *unimodular* (constant modulus), and hence, the BER performance of the MIMO-OFDM system will not be affected by this precoding. In other words, assume a unimodular diagonal precoding matrix $\boldsymbol{A} \in \mathbb{C}^{MN_t \times MN_t}$ is premultiplied with the data matrix $\boldsymbol{D}$. Then, instead of the original MIMO-OFDM signal $\boldsymbol{Y}$, we construct a desired transmit matrix $\boldsymbol{S}$ as

$$\boldsymbol{S} = \boldsymbol{W}^H \boldsymbol{A} \boldsymbol{D} \boldsymbol{F}^H \tag{7}$$

Now, the PAPR reduction problem reduces to the following program

$$\min_{\boldsymbol{a}} \quad \|\mathrm{vec}(\boldsymbol{S})\|_\infty^2 \quad \text{s.t.} \quad \|\mathrm{vec}(\boldsymbol{S})\|_2^2 = P, \tag{8}$$

where $\boldsymbol{a} = \mathrm{vecdiag}(\boldsymbol{A})$. Note that the program of (8) is not convex due to the non-linear equality constraint. The above non-convex program can be further simplified via using the properties of Kronecker product and noting that $\mathrm{vec}(\boldsymbol{S}) = (\boldsymbol{B}^T \circ \boldsymbol{W})\mathrm{vecdiag}(\boldsymbol{A}) = \boldsymbol{C}\boldsymbol{a}$, where $\boldsymbol{C} \in \mathbb{C}^{MN_t \times N}$, $(\cdot)^T$ denotes the matrix transpose, $\circ$ denotes the Khatri-Rao product, and the function $\mathrm{vecdiag}(\cdot)$ forms a column vector whose elements are the main diagonal of the matrix argument. In [8], the authors has considered the following equivalent optimization problem:

$$\min_{\boldsymbol{a}} \quad \|\boldsymbol{C}\boldsymbol{a}\|_\infty^2 \quad \text{s.t.} \quad \|\boldsymbol{C}\boldsymbol{a}\|_2^2 = \alpha N_T, \tag{9}$$

where αN_T is a fixed total transmit power. The above program is not convex and admits many local minimum and is generally hard to tackle. In the next section, we propose a novel unimodular quadratic programming approach based on the power method-like iteration [10] which is guaranteed to converge to good solutions, and also is computationally efficient, and thus, making it a suitable candidate for real-time signal processing applications in MIMO-OFDM systems.

III. The Proposed Algorithm

A. k-Modular Quadratic Programming (k-MQP)

In its general form, the PAPR reduction problem in our setting can be formulated as follows:

$$\begin{aligned} \min_{\boldsymbol{a}} \quad & \|\boldsymbol{C}\boldsymbol{a}\|_\infty^2 \\ \text{s.t.} \quad & |\boldsymbol{a}(n)| \le k, \ \forall n, \\ & \|\boldsymbol{a}\|_2^2 = MN_t, \end{aligned} \tag{10}$$

where $\boldsymbol{a}(n)$ denotes the n-th element of the vector $\boldsymbol{a}$. The main idea in [8], is to replace the infinity norm in (9) with the average deviation of the signal from a CM signal with the hope of obtaining the resulting signal $\boldsymbol{S}$ close to a CM signal, and considering that the PAPR achieves its minimum for such a signal. Namely, instead of the objective function in (9), the authors consider the following cost function

$$\boldsymbol{J}_1(\boldsymbol{a}) = \|\boldsymbol{C}\boldsymbol{a} \odot (\boldsymbol{C}\boldsymbol{a})^* - \alpha \mathbf{1}_{N_t}\|_2^2 = \sum_{n=1}^{N_T} \left(\boldsymbol{a}^H \boldsymbol{c}_n \boldsymbol{c}_n^H \boldsymbol{a} - \alpha\right)^2, \quad (11)$$

where $\boldsymbol{c}_n$ denotes the n-th row of matrix $\boldsymbol{C}$, $\mathbf{1}_{N_T}$ represents the all-one vector with dimension N_T, and $\odot$ denotes the point-wise multiplication. The objective function $\boldsymbol{J}_1(\boldsymbol{a})$ appears to be a *quartic* function of $\boldsymbol{a}$. However, one can verify that the objective function of (9) is *almost equivalent* to a quadratic function; see the following. We first note that

$$\boldsymbol{J}_1(\boldsymbol{a}) = \sum_{n=1}^{N_T} \left(|\boldsymbol{c}_n^H \boldsymbol{a}|^2 - \alpha\right)^2. \quad (12)$$

Briefly speaking, $\boldsymbol{J}_1(\boldsymbol{a})$ achieves small values (or becomes zero) if and only if

$$\boldsymbol{J}_2(\boldsymbol{a}) = \min_{\{\phi_n\}} \underbrace{\sum_{n=1}^{N_T} |\boldsymbol{c}_n^H \boldsymbol{a} - \sqrt{\alpha} e^{j\phi_n}|^2}_{\boldsymbol{J}_3(\boldsymbol{a}, \{\phi_n\})}, \quad (13)$$

becomes small (or zero) (for a detailed analysis see [10], [11]), where the set of phases $\{\phi_n\}_{n=1}^{N_T}$ are exploited as auxiliary variables. Alternatively, one can rewrite $\boldsymbol{J}_2(\boldsymbol{a})$ as:

$$\boldsymbol{J}_2(\boldsymbol{a}) = \max_{\{\phi_n\}} \underbrace{\sum_{n=1}^{N_T} |\boldsymbol{c}_n^H \boldsymbol{a} + \sqrt{\alpha} e^{j\phi_n}|^2}_{\boldsymbol{J}_3(\boldsymbol{a}, \{\phi_n\})}. \quad (14)$$

When the latter $\boldsymbol{J}_2(\boldsymbol{a})$ achieves its maximal value, $\boldsymbol{J}_1(\boldsymbol{a})$ will also assume small value (or zero). The next point to note is that the criterion in (14) can be alternatively expressed as

$$\begin{aligned} \boldsymbol{J}_3(\boldsymbol{a}, \{\phi_n\}) &= \sum_{n=1}^{N_T} \left(\boldsymbol{c}_n^H \boldsymbol{a} + \sqrt{\alpha} e^{j\phi_n}\right)^H \left(\boldsymbol{c}_n^H \boldsymbol{a} + \sqrt{\alpha} e^{j\phi_n}\right) \\ &= \sum_{n=1}^{N_T} \left[\boldsymbol{a}^H \boldsymbol{c}_n \boldsymbol{c}_n^H \boldsymbol{a} + 2\Re\{\sqrt{\alpha} \boldsymbol{a}^H \boldsymbol{c}_n e^{j\phi_n}\} + \alpha\right]. \end{aligned} \quad (15)$$

Therefore, the maximization of $\boldsymbol{J}_3(\boldsymbol{a}, \{\phi_n\})$ with respect to either $\boldsymbol{a}$ or $\{\phi_n\}$ can be equivalently written in terms of a new objective function $\boldsymbol{J}_4(\boldsymbol{a}, \{\phi_n\})$ as follows

$$\max_{\boldsymbol{a}, \{\phi_n\}} \quad \boldsymbol{J}_4(\boldsymbol{a}, \{\phi_n\}) \quad \text{s.t.} \quad \boldsymbol{a}, \{\phi_n\} \in \Omega, \quad (16)$$

where Ω denotes the search space of the optimization variables, and

$$\boldsymbol{J}_4(\boldsymbol{a}, \{\phi_n\}) = \begin{pmatrix} \boldsymbol{a} \\ 1 \end{pmatrix}^H \underbrace{\begin{pmatrix} \boldsymbol{C}' & \boldsymbol{b} \\ \boldsymbol{b}^H & \alpha \end{pmatrix}}_{\tilde{\boldsymbol{C}}} \begin{pmatrix} \boldsymbol{a} \\ 1 \end{pmatrix}, \quad (17)$$

where,

$$\boldsymbol{C}' = \sum_{n=1}^{N_T} \boldsymbol{c}_n \boldsymbol{c}_n^H = \boldsymbol{C}^H \boldsymbol{C}, \quad \boldsymbol{b} = \sqrt{\alpha} \sum_{n=1}^{N_T} \boldsymbol{c}_n e^{j\phi_n}.$$

TABLE I

THE PROPOSED OPTIMIZATION ALGORITHM FOR PAPR REDUCTION

Step 0: Initialize the precoding vector $\boldsymbol{a} \in \mathbb{C}^{MN_t}$ with a unimodular (or low PAPR) vector. Choose a desirable PAPR constraint value k, i.e., set $k = 1$ for obtaining a unimodular solution or set $k > 1$ to obtain a more general PAPR.
Step 1: Compute the matrix $\boldsymbol{C} = (\boldsymbol{B}^* \circ \boldsymbol{W})^H$ and form the matrix $\tilde{\boldsymbol{C}}$ as defined in (17).
Step 2: Employ the power method-like iterations following (20) or (22) (depending on the PAPR constraint δ) to update $\boldsymbol{a}$.
Step 3: Repeat Step 2 until a pre-defined outer-loop iteration number is satisfied.

The maximization of $\boldsymbol{J}_4$ in (16) can be tackled via employing a cyclic optimization approach with respect to $\boldsymbol{a}$ and $\{\phi_n\}$. Note that for fixed $\boldsymbol{a}$, the maximizers of $\boldsymbol{J}_4$ are simply given by:

$$\phi_n = \arg\left(\boldsymbol{c}_n^H \boldsymbol{a}\right), \quad (18)$$

where, $\arg(\cdot)$ denotes the phase angle operator, and more generally, in its vectorized form as

$$\boldsymbol{\phi} := [\phi_1, \ldots, \phi_n]^T = \arg(\boldsymbol{C}\boldsymbol{a}). \quad (19)$$

On the other hand, for fixed $\boldsymbol{\phi}$, the optimization of $\boldsymbol{J}_4$ with respect to $\boldsymbol{a}$ boils down to

$$\begin{aligned} \max_{\boldsymbol{a}} \quad & \begin{pmatrix} \boldsymbol{a} \\ 1 \end{pmatrix}^H \tilde{\boldsymbol{C}} \begin{pmatrix} \boldsymbol{a} \\ 1 \end{pmatrix} \\ \text{s.t.} \quad & |\boldsymbol{a}(n)| \leq k, \forall n, \\ & \|\boldsymbol{a}\|_2^2 = MN_t, \end{aligned} \quad (20)$$

The optimization problem in (20) can be tackled efficiently using the power method-like iterations introduced and discussed extensively in [10], [12], and [13]. We notice that $\tilde{\boldsymbol{C}}$ is always positive definite, then for the unimodular case (i.e., $k = 1$), the objective function in (20) can be monotonically increased via the following iterations:

$$\boldsymbol{a}^{(s+1)} = \eta\left(\exp\left(j\arg\left(\tilde{\boldsymbol{C}}\begin{pmatrix} \boldsymbol{a}^{(s)} \\ 1 \end{pmatrix}\right)\right)\right), \quad (21)$$

where s denotes the iteration number, and $\eta(\cdot)$ takes the first MN_t entries of the vector argument. For the case of $k > 1$, the objective function in (20) can be monotonically increased via considering the following nearest-vector problem at each iteration:

$$\begin{aligned} \min_{\boldsymbol{a}^{(s+1)}} \quad & \left\|\boldsymbol{a}^{(s+1)} - \eta\left(\hat{\boldsymbol{C}}\begin{pmatrix} \boldsymbol{a}^{(s)} \\ 1 \end{pmatrix}\right)\right\|_2^2, \\ \text{s.t.} \quad & |\boldsymbol{a}^{(s+1)}(n)| \leq k, \forall n, \\ & \|\boldsymbol{a}^{(s+1)}\|_2^2 = MN_t. \end{aligned} \quad (22)$$

The latter problem can be tackled efficiently using an $O(MN_t)$ recursive algorithm in [14].

The proposed optimization algorithm for PAPR reduction derived above is summarized in Table I.

IV. Numerical Results

The comparison of choosing different PAPR constraint values k are evaluated as follows in 10MHZ WiMAX system. Each RB spans 14 sub-carriers over two OFDM symbols in time, containing 24 data symbols and 4 pilot symbols. The simulation system possesses 2 transmitting antennas and 2 receiving antennas. there are total of 60 RBs, including $M_t N_d = 840$ data subcarriers with QPSK modulation. 10,000 OFDM blocks are randomly generated and for each of them, a random complex fading channel is generated, and the beamforming matrices $\boldsymbol{W}$ are chosen as the right singular vectors of these matrices.

A. Performance in PAPR Reduction

In Fig. 1, Complementary cumulative distribution functions (CCDF) curves are shown for k-MQP (20 iterations) with various choices of k, compared to UC-CMA [8] (20 iterations). When we set $k \geqslant 2$, the PAPR reduction performance of k-MQP are very close. In the case of k-MQP ($k \geqslant 2$), PAPR reduction of up to 7.3 dB. 1-MQP is worse by about 1.4 dB. Moreover, comparing to UC-CMA, CCDF curves show superior performance of k-MQP in almost 70% of OFDM blocks in setting $k \geqslant 2$.

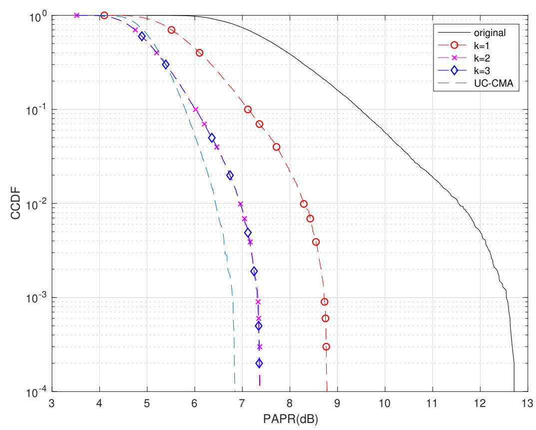

Fig. 1. PAPR reduction performance of k-MQP and UC-CMA

B. Bit Error Rate

Fig. 2 shows BER versus SNR curves for the QPSK-OFDM system in scenarios that k-MQP and UC-CMA weights are applied at the transmitter. The channel in the case of both approaches is AWGN and the received vector is divided by $\boldsymbol{a}$ to equalize the PAPR weights. As expected, 1-MQP and UC-CMA does not effect the BER performance and perfect channel recovery is assumed when $k \geqslant 2$.

C. Convergence rate comparison

With iteration numbers growing, 1,000 OFDM blocks are randomly chosen to record the PAPR reduction value. As illuminated in Fig. 3, obviously, the k-MQP ($k \geqslant 2$) algorithm converges faster. Average PAPR value for k-MQP ($k \geqslant 2$) is lower about 0.15dB than the one for UC-CMA.

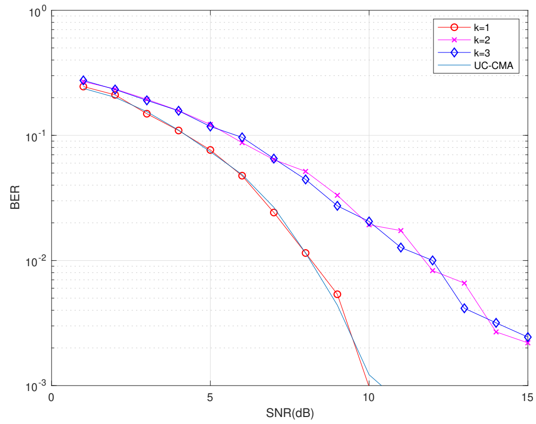

Fig. 2. BER performance comparison of k-MQP and UC-CMA

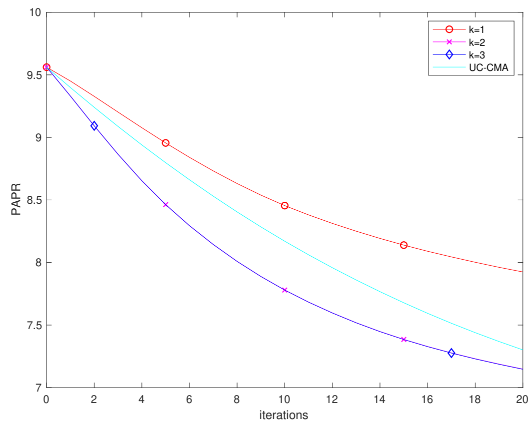

Fig. 3. comparison of average PAPR values versus outer-loop iteration numbers for k-MQP and UC-CMA

V. Concluding Remarks

In this paper, we proposed a QP formulation for PAPR reduction in OFDM systems and tackled the problem by employing the power method-like iterations. . The main results can be summarized as follows:

- The proposed algorithm considers a doubly PAPR constrained scenario in OFDM systems, which can handle arbitrary PAPR constraints and enjoys from good convergence rate (i.e., suitable candidate for real-time signal processing applications)
- At the expense of N_t auxiliary variables (which their optima are analytically given), we can introduce a quadratic alternative for the quartic cost function ($\boldsymbol{J}_1$) and the ℓ_∞-norm based objective in (10). Undertaking such approach paves the way for tackling the problem more easily as QPs are more widely studied.
- The power method-like iterations discussed in (20)-(22) are studied previously, and are proved to converge locally, with a monotonic change in the objective function. The use of such iterations can be extended to M-ary alphabet or other kinds of structured signals.

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